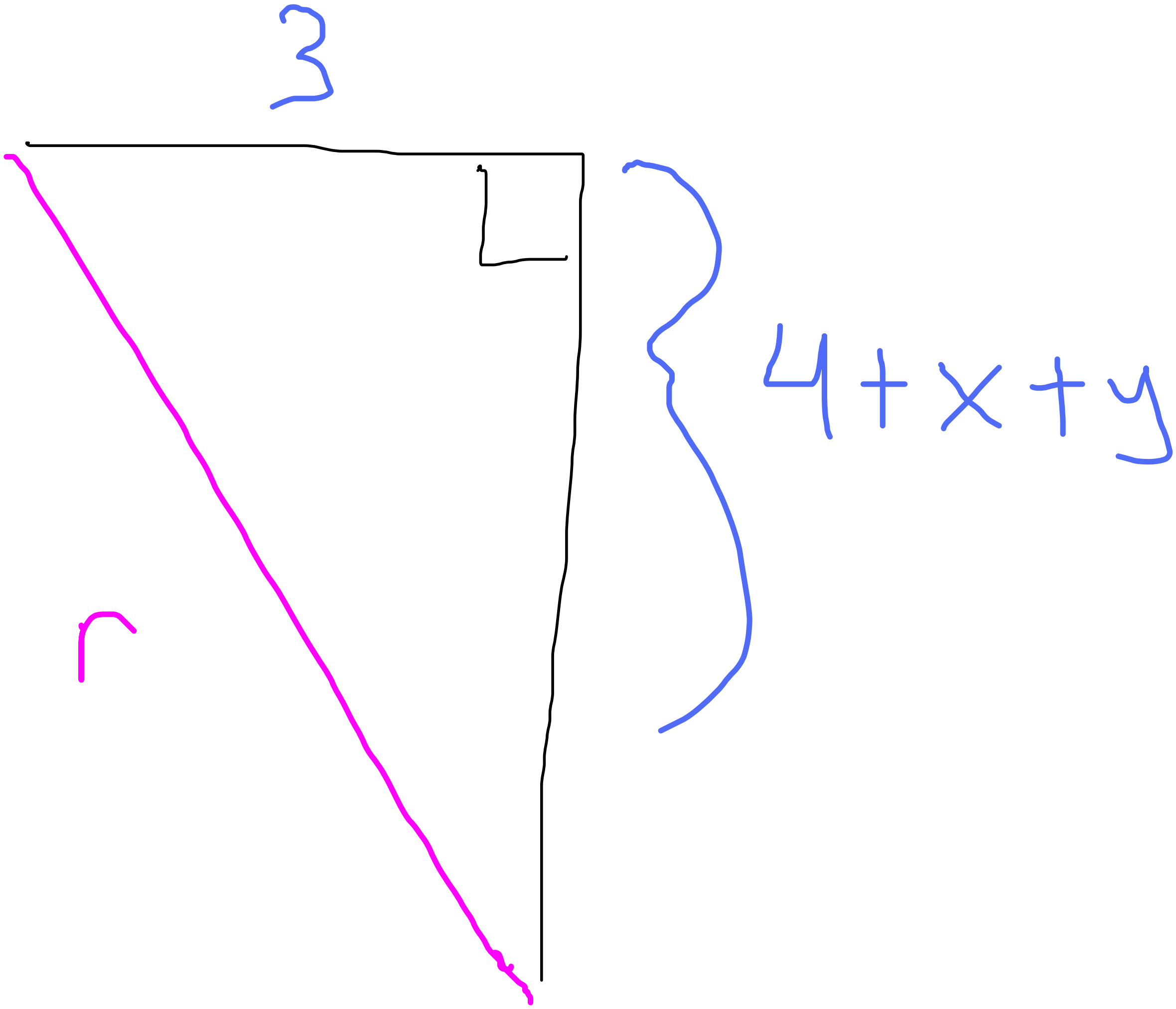
Pythagorean Theorem

Want: $\frac{dr}{dt}$



$$(4+x+y)^2 + 3^2 = r^2$$

$$(4+x+y)^2 + 9 = r^2$$

Differentiate

$(4+x+y)^2 + 9 = r^2$ with

respect to t.

$$2(4+x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2r\frac{dr}{dt}$$

Know $\frac{dx}{dt} = 20$ mph

$\frac{dy}{dt} = 15$ mph (positive)

Want $\frac{dr}{dt}$ at $t = 3$ hours.

$$\cancel{2}(4+x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right) = \cancel{2}r\frac{dr}{dt}$$

$$(4+x+y)(20+15) = r\frac{dr}{dt}$$

Want $\frac{dr}{dt}$ at $t=3$, this

will help us find x, y, and r.

x = distance Bigfoot travels

$= (\text{rate}) \cdot (\text{time})$

$= (20 \text{ mph})(3 \text{ h}) = 60 \text{ m.}$

y = distance Sasquatch travels

$= (15)(3) = 45 \text{ m.}$

Then $r^2 = (4+x+y)^2 + 9$

$$= (4+60+45)^2 + 9$$

$$= (109)^2 + 9$$

$$= 11{,}881 + 9$$

$$= 11{,}890 \text{ mi}^2$$

So $r = \sqrt{11{,}890}$ mi.

Plug all these numbers into the equation for $\frac{dr}{dt}$

$$(4+x+y)(35) = r\frac{dr}{dt}$$

$$(4+60+45)\cdot(35) = \sqrt{11{,}890}\,\frac{dr}{dt}$$

$$3815 = \sqrt{11{,}890}\,\frac{dr}{dt}$$

so $\boxed{\dfrac{dr}{dt} = \dfrac{3815}{\sqrt{11{,}890}} \text{ mph}}$

Strategy For Solving Related Rate Problems

1) Draw a picture

2) Label picture

3) Find a relationship between the unknown (last line of story problem) and the other variables

4) Differentiate (always implicit, usually with to time)

5) Plug in all given numbers, solve for the unknown.

Only plug in numbers for your variables AFTER differentiation.

Example 1: A satellite is stationed 350 km above the CERN in Switzerland. A neutrino is shot from CERN to INFN 730 km away. Suppose the neutrino and the satellite are moving in the same direction and the satellite's speed is 7.68 km/s. The satellite tracks the speed of the neutrino.

a) If the satellite measures that the neutrino covered the distance in 0024349571 seconds, how fast was the neutrino travelling?

b) How fast is the angle of elevation between the neutrino and the satellite changing .001 seconds after the experiment begins?

c) Is the neutrino really that fast?

a) $d = rt$

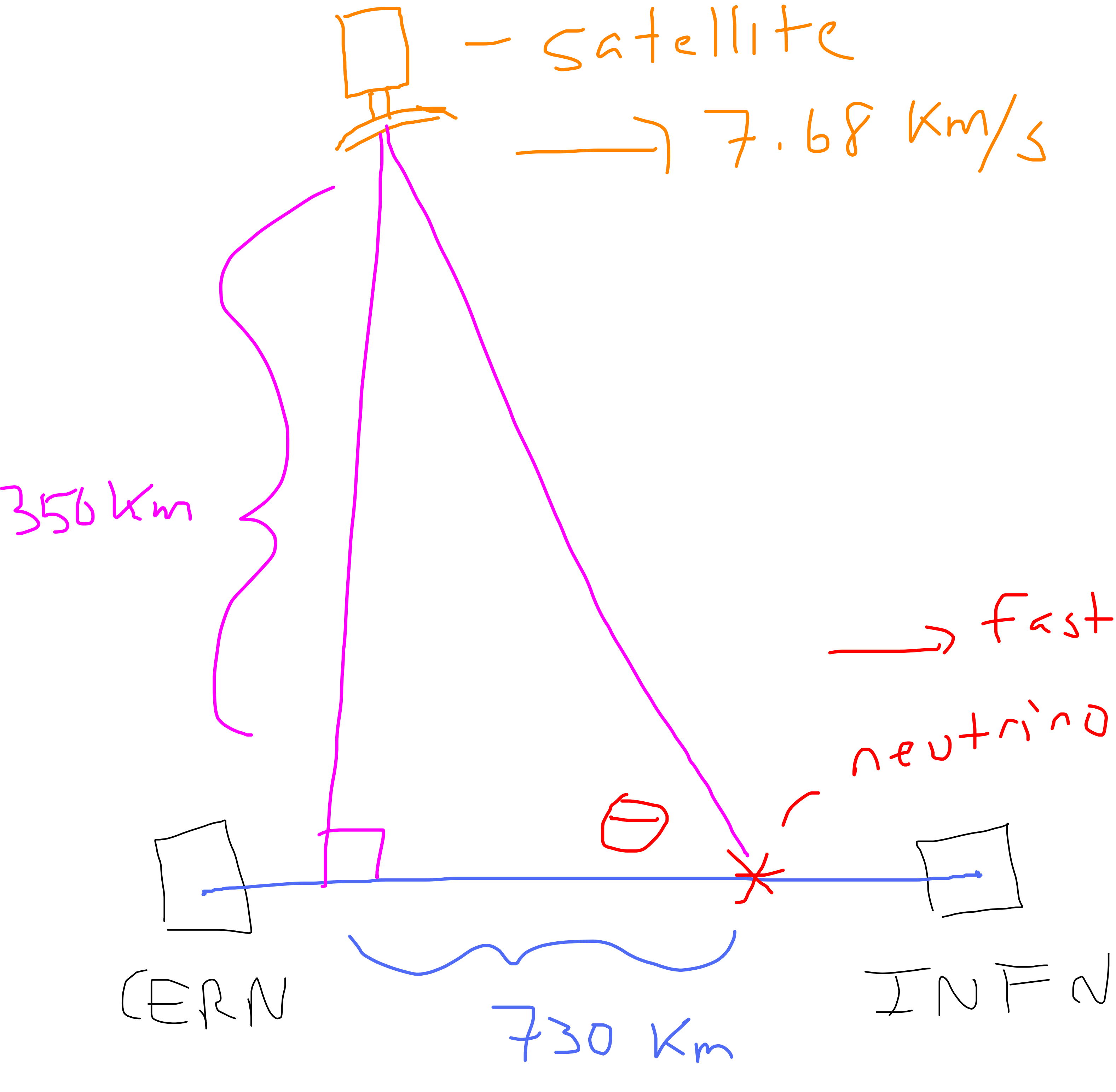
$$r = \frac{d}{t}$$

$$= \frac{730 \text{ km}}{.0024349571 \text{ s}}$$

$$= 299799.955 \text{ km/s}$$

Speed of light

$$= 299792.458 \text{ km/s}$$



Want $\frac{d\theta}{dt}$

Let x = base of triangle

$$\tan\theta = \frac{350}{x} \leftarrow \text{relationship}$$

Differentiate wrt time.

$$\frac{d}{dt}\left(\tan(\theta)\right) = \frac{d}{dt}\left(\frac{350}{x}\right)$$

$$\underbrace{\sec^2\theta \frac{d\theta}{dt}}_{\text{Chain rule}} = 350 \frac{d}{dt}\left(x^{-1}\right)$$

$$= \underbrace{350\left(-x^{-2}\right)\frac{dx}{dt}}_{\text{Chain rule}}$$

$$\sec^2(\theta)\frac{d\theta}{dt} = 350\left(-\frac{1}{x^2}\right)\frac{dx}{dt}$$

$$\frac{}{\sec^2(\theta)} \qquad \frac{}{\sec^2\theta}$$

$$\boxed{\frac{350}{\sec^2(\theta)} = \frac{350}{\left(\frac{1}{\cos^2(\theta)}\right)} = \cos^2(\theta)350}$$

$$\frac{d\theta}{dt} = -\frac{350}{x^2}\cos^2(\theta)\frac{dx}{dt}$$

$$\frac{dx}{dt} = (\text{speed of neutrino}) - (\text{speed of satellite})$$

$= 299{,}799{,}955\ \text{km/s} - 7.68\ \text{km/s}$

$= 299{,}792{,}275\ \text{km/s}$ (KC)

$$x = \frac{dx}{dt} \cdot t$$

$$= (299{,}792{,}275 \text{ km/s})(.0015)$$

$$= 299.792275 \text{ km}$$

To find $\cos\theta$, remember

that $\cos\theta = \frac{adj}{hyp}$

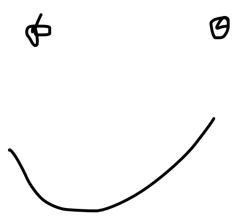
$$= \frac{x}{hyp}$$

$$hyp^2 = x^2 + 350^2$$

$$= (299.792\ 275)^2 + 350^2$$

$$\frac{d\theta}{dt} = \left(\frac{\cancel{(299{,}792275)^2}}{(299{,}792275)^2 + (356)^2} \right) \frac{-356}{\cancel{x^2}}$$

$$= 299{,}792{,}275$$

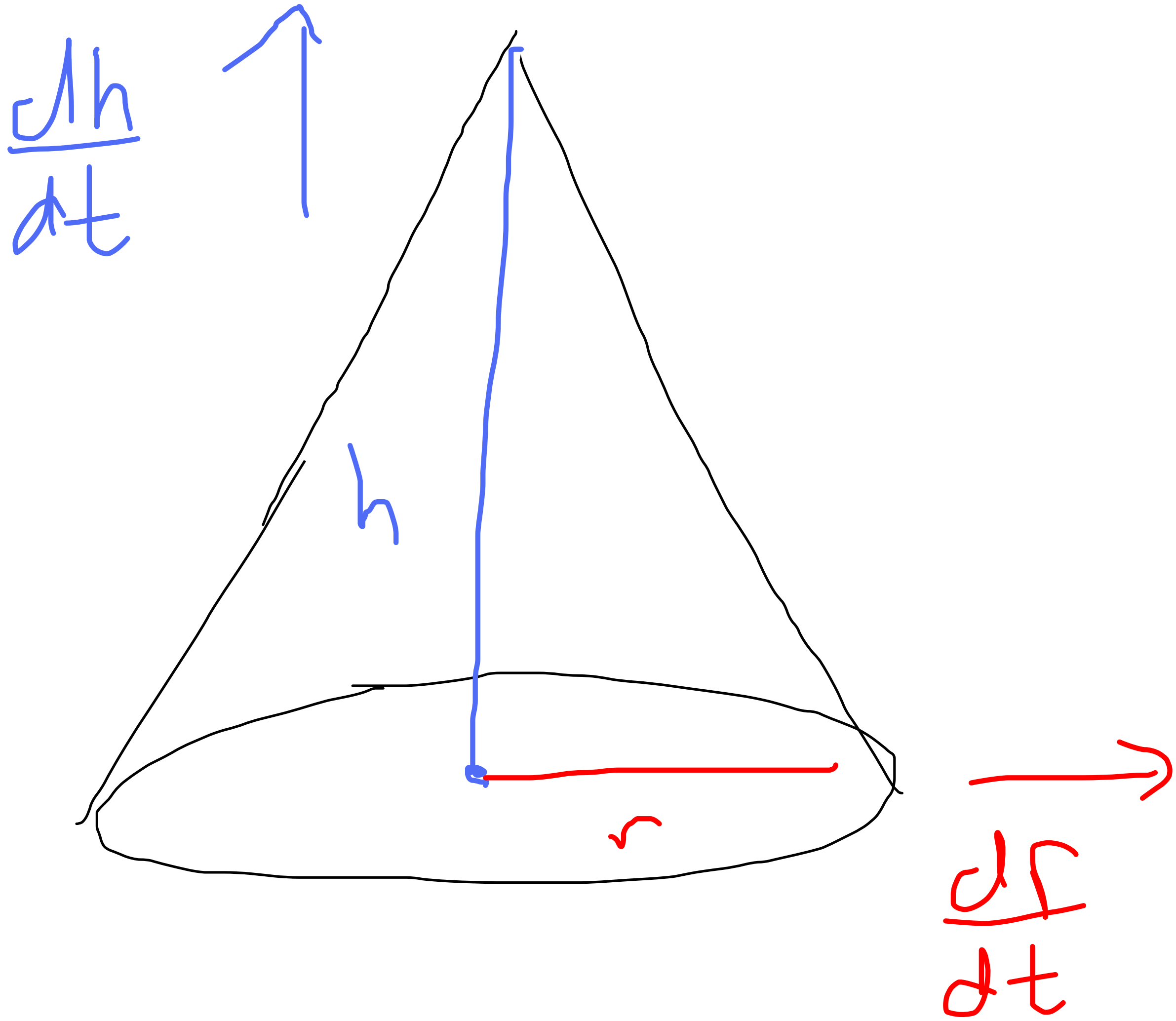
$$=$$  km/s

c) If you take into account the speed of the satellite, the answer appears to be that the neutrino is slower than light.

#27, 2, 8.

Example 4: Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min; and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height increasing when the pile is 10 ft high?

Picture



We know that the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h.$$ Given that $2r = h$,

$\boxed{h = 2r}$, which gives us

relationship

$$V = \frac{1}{3}\pi r^2 \cdot 2r$$

$$= \frac{2}{3}\pi r^3$$

Differentiate with respect to t,

chain rule

$$\frac{dV}{dt} = \frac{2}{3}\pi \overbrace{3r^2\frac{dr}{dt}}$$

$\frac{dV}{dt} = 30\ \frac{ft^3}{min}$, $h = 10\,ft$, $r = 5\,ft$.

$$30 = \frac{\cancel{2}}{\cancel{3}} \pi \cdot \cancel{3} (5^2) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{30}{50\pi} \text{ ft/min}$$

$$= \boxed{\frac{3}{5\pi} \text{ ft/min}}$$